International Research Journal of Education and Technology
Peer Reviewed Journal
ISSN 2581-7795

On the non-homogeneous cubic diophantine equation with five unknowns

$$x\,y - z\,w = R^3$$

S.Vidhyalakshmi[1], M.A.Gopalan[2]

[1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,Trichy-620 002,Tamil Nadu, India.

[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

Abstract:

This paper illustrates the process of determining non-zero distinct integer solutions to the non-homogeneous cubic equation with five unknowns $x\,y - z\,w = R^3$. A few relations between the solutions and special figurate numbers are presented.

Keywords : non-homogeneous cubic , cubic with five unknowns ,integer solutions ,

Special figurate numbers

Notations:

$$P_k^3 = \frac{k(k+1)(k+2)}{6}, P_k^5 = \frac{k^2(k+1)}{2}, t_{m,n} = n(1+\frac{(n-1)(m-2)}{2}), CP_k^6 = k^3,$$

$$CP_k^8 = \frac{8k^3 - 2k}{6} \quad CP_k^{18} = 3k^3 - 2k, CP_k^{12} = 2k^3 - k, CP_k^{16} = \frac{8k^3 - 5k}{3},$$

Introduction:

The Diophantine equations are rich in variety and offer an unlimited field for research . In particular refer [1-16] for a few problems on cubic equation with five unknowns. This paper concerns with yet another interesting cubic diophantine equation with five variables given by $xy - zw = R^3$ for determining its infinitely many non-zero distinct integral solutions.A few relations between the solutions and special figurate numbers are presented.

International Research Journal of Education and Technology
Peer Reviewed Journal
ISSN 2581-7795

Method of analysis :

The non-homogeneous cubic equation with five unknowns to be solved is

$$xy - zw = R^3 \tag{1}$$

The introduction of the transformations

$$x = u + v, y = u - v, z = p + v, w = p - v, u \neq v \neq p \tag{2}$$

in (1) leads to

$$u^2 - p^2 = R^3 \tag{3}$$

Solving (3) in different ways , the values of u, p, R are obtained. In view of (2) , the corresponding values of x, y, z, w satisfying (1) are determined.

Way 1 :

Employing the well-known identity

$$(a+b)^2 - (a-b)^2 = 4ab, \tag{*}$$

it is seen that (3) is satisfied by

$$u = \frac{R^3 + 1}{2}, p = \frac{R^3 - 1}{2}$$

As our interest is on finding integer solutions ,observe that the choice

$$R = 2k + 1 \tag{4}$$

gives

$$u = 4k^3 + 6k^2 + 3k + 1, p = 4k^3 + 6k^2 + 3k$$

In view of (2), it is seen that

$$\left. \begin{array}{l} x = 4k^3 + 6k^2 + 3k + 1 + v, y = 4k^3 + 6k^2 + 3k + 1 - v, \\ z = 4k^3 + 6k^2 + 3k + v, w = 4k^3 + 6k^2 + 3k - v \end{array} \right\} \tag{5}$$

Thus,(4) and (5) represent the integer solutions to (1).

Relations between the solutions and special figurate numbers:

(I). Each of the following expressions is a cubical integer:

$$x+w,\, y+z,\, x+y-1,\, z+w+1$$

(II). $x+y-6P_k^3-14P_k^5$ is a square multiple of 2

(III). $z+w-3CP_k^{16}-t_{26,k}\equiv 0 \pmod{22}$

(IV). $z+w-6CP_k^{8}-t_{26,k}\equiv 0 \pmod{19}$

Note 1:

The other choices of integer solutions to (1) on using (*) are exhibited below:

Choice 1:

$$x=t_{3,k}+v,\, y=t_{3,k}-v,\, z=t_{3,k-1}+v,\, w=t_{3,k-1}-v,\, R=k$$

Choice 2:

$$x=s^{2\alpha}+2s^{\alpha}+v,\, y=s^{2\alpha}+2s^{\alpha}-v,\, z=s^{2\alpha}-2s^{\alpha}+v,\, w=s^{2\alpha}-2s^{\alpha}-v,\, R=2s^{\alpha}$$

Choice 3:

$$x=2s^{2\alpha}+s^{\alpha}+v,\, y=2s^{2\alpha}+s^{\alpha}-v,\, z=2s^{2\alpha}-s^{\alpha}+v,\, w=2s^{2\alpha}-s^{\alpha}-v,\, R=2s^{\alpha}$$

Choice 4:

$$x=2s^{3\alpha}+1+v,\, y=2s^{3\alpha}+1-v,\, z=2s^{3\alpha}-1+v,\, w=2s^{3\alpha}-1-v,\, R=2s^{\alpha}$$

Way 2:

Taking

$$R=p$$

in (3) , it gives

$$u^2=p^2(1+p)$$

which is satisfied by

$$p=k^2+2k,\, u=(k^2+2k)(k+1)$$

Using (2),the corresponding integer solutions to (1) are given by

$$x=(k^2+2k)(k+1)+v,\, y=(k^2+2k)(k+1)-v,$$
$$z=k^2+2k+v,\, w=k^2+2k-v,\, R=k^2+2k$$

Relations between the solutions and special figurate numbers :

(i). $x+y=12P_k^3$

(ii). $x + w - 2P_k^5 - t_{8,k} \equiv 0 \pmod 6$

(iii). $x + w = CP_k^6 + 8t_{3,k}$

(iv). $x + w - CP_k^{18} - t_{26,k} \equiv 0 \pmod{25}$

(v). $z + w - t_{6,k} \equiv 0 \pmod 5$

(vi). $x + y = 4P_k^5 + 8t_{3,k}$

(vii). $x + y - CP_k^{12} - t_{14,k} \equiv 0 \pmod{10}$

(viii). $x + y - CP_k^{12} - 10t_{3,k}$ is a perfect square

Note 2:

Suppose one assumes

$$R = u$$

in (3), then the corresponding integer solutions to (1) are given by

$$x = -(k^2 + 2k) + v, \; y = -(k^2 + 2k) - v,$$
$$z = -(k^2 + 2k)(k+1) + v, \; w = -(k^2 + 2k)(k+1) - v, \; R = -(k^2 + 2k)$$

Conclusion:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with five unknowns given by $xy - zw = R^3$.One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables.

References :

1. M.A.Gopalan ,S.Vidhyalakshmi and T.R.Usha Rani ,On the cubic equation with five unknowns $x^3 + y^3 = z^3 + w^3 + t^2(x + y)$ Indian Journal of Science ,Vol.1 ,No.1 ,17-20,2012

2. M.A.Gopalan ,S.Vidhyalakshmi and T.R.Usha Rani ,Integral solutions of the cubic equation with five unknowns $x^3 + y^3 + u^3 + v^3 = 3t^3$,IJAMA ,Vol.4(2) ,147-151 ,2012

3. M.A.Gopalan, S. Vidhyalakshmi, M.Manjula, E. Premalatha and N.Thiruniraiselvi, On homogeneous cubic equation with five unknowns $x^3 + y^3 = z(P^2 + W^2 - PW)$, Cayley J.Math,

Vol.2(2),Pp.131-137, 2013

4. M.A.Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi, On the cubic equation with five Unknowns $3(x^3-y^3)=z^3-w^3+12t^2+4$, International Journal of Engineering , Science and Mathematics, Vol.2(1),Pp.227-236, March 2013

5.M.A.Gopalan and V.Geetha ,Integral solutions of the homogeneous cubic equation $x^3+y^3+xy(x+y)-z^3-w^3-zw(z+w)=(x+y+z+w)X^2$,International Journal of Computational Engineering Research ,Vol.3(5),23-27,2013

6. M.A. Gopalan, S. Vidhyalakshmi, S. Mallika, " Integral points on the cubic equation with five unknowns $\mathbf{x}^3+\mathbf{y}^3=\mathbf{z}^3+\mathbf{w}^3+6\mathbf{t}^2$", Diophantine J. Math., Volume 2, No.1, 39-46, 2013.

7. M.A. Gopalan, S. Vidhyalakshmi, T.R. Usharani, " Integral solutions on the cubic equation with five unknowns $\mathbf{x}^3+\mathbf{y}^3+\mathbf{u}^3+\mathbf{v}^3=\mathbf{kt}^3$", Bessel J. Math., Volume 3, No. 1, 69-75, 2013.

8. M.A. Gopalan, S. Vidhyalakshmi, S. Mallika, " Integral points on the cubic equation with five unknowns $\mathbf{x}^3+\mathbf{y}^3=\mathbf{z}^3+\mathbf{w}^3+6(\mathbf{x}+\mathbf{y})+6\mathbf{t}^2$", AJAMMS, Volume 2, No.1, 31-35, 2013.

9. M.A. Gopalan, S. Vidhyalakshmi, A. Kavitha, " On the cubic equation with five unknowns $4(\mathbf{x}^3+\mathbf{y}^3)=\mathbf{z}\left[4\mathbf{w}^2+4\mathbf{p}^2-4\mathbf{pw}+(\mathbf{x}+\mathbf{y})^2\right]$", Archimedes J. Math., 4(1), 19-25, 2014.

10.S. Vidhyalakshmi, M.A. Gopalan, K. Lakshmi," On the non-homogeneous cubic equation with five unknowns $\mathbf{x}^2+\mathbf{xy}-\mathbf{y}^2-\mathbf{z}-\mathbf{w}=\mathbf{T}^3$", IRJMEIT, Volume 1, Issue 4, 1-8, 2014.

11. M.A.Gopalan ,V.Geetha and P.Geethanjali ,On the cubic Diophantine equation with five unknowns $x^3+y^3+(x+y)z^3=w^2-R^2$,Jamal Academic Research Journal-an interdisciplinary ,special issue,259-265 ,2014

12.M.A.Gopalan ,S.Vidhyalakshmi ,V.Geetha ,M.Kalaimathi and V.Krithika ,On the cubic diophantine equation with five unknowns $x^3+y^3+(x+y)(x-y)^2=16(z+w)p^2$, International Journal of Physics and Mathematical Sciences ,Vol .5(1) ,59-63 ,2015

13.M.A.Gopalan and V.Geetha ,On the non-homogeneous cubic equation with five unknowns $6(x^3-y^3)=z^3-w^3+12p^2+10$, Jamal Academic Research Journal-an interdisciplinary, special issue,274-279 ,2015

14. M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, " On the non-homogeneous cubic equation with five unknowns $9(\mathbf{x}^3-\mathbf{y}^3)=\mathbf{z}^3-\mathbf{w}^3+12\mathbf{p}^2+16$", IJIRR, Volume 3, Issue 6, 2525-2528, 2016.

15. M.A. Gopalan, J. Shanthi," On the non-homogeneous cubic equation with five unknowns $(\mathbf{a}+1)^2(\mathbf{x}^3-\mathbf{y}^3)=(2\mathbf{a}+1)(\mathbf{z}^3-\mathbf{w}^3)+6\mathbf{a}^2\mathbf{p}^2+2\mathbf{a}^2$ ", IJMSET, Volume 3, Issue 5, 32-36, 2016.

16. M.A.Gopalan,S.Vidhyalakshmi and S.Hemalatha ,On The Cubic Equation with Five Unknowns , $x^3+y^3=84(z+w)p^2$,AJAST ,Vol.1,Issue 7 ,144-146 ,2017